

# Contrôle des EDPs : approches en mathématique et en automatique

Programme et résumés

## Programme

|               | Mercredi 2 novembre | Jeudi 3 novembre       |
|---------------|---------------------|------------------------|
| 9h30 – 10h30  | Christophe Prieur   | Yann Le Gorrec         |
| 10h30 – 11h00 | <i>Pause café</i>   | <i>Pause café</i>      |
| 11h00 – 11h45 | Sylvain Ervedoza    | Frédéric Marbach       |
| 11h45 – 12h30 | Nicolas Espitia     | Lucie Baudouin         |
| 12h30 – 14h30 | <i>Déjeuner</i>     | <i>Déjeuner</i>        |
| 14h30 – 15h30 | Marius Tucsnak      | Jean-Michel Coron      |
| 15h30 – 16h00 | <i>Pause café</i>   | <i>Pause café</i>      |
| 16h00 – 16h45 | Ying Tang           | Delphine Bresch-Pietri |
| 16h45 – 17h15 | <i>Pause</i>        | <i>Pause</i>           |
| 17h15 – 18h00 | Hoai-Minh Nguyen    | Vincent Andrieu        |

## Résumés

### Global exponential set-point regulation for abstract Cauchy problems with input saturation

*Vincent Andrieu*

Output tracking is one of the primary goals of control feedback and is usually handled by an integral controller. In this talk, we discuss how to construct Lyapunov functional associated with this problem and how this can be employed for some nonlinear operators. In a second step, we consider saturation of the control input. It is shown how global exponential stability can still be obtained by adding an anti-windup loop to the integral action.

### Contrôle basé événement d'un système de dimension infinie

*Lucie Baudouin*

Ou comment préserver la stabilité exponentielle malgré l'échantillonnage temporel de la commande. Prenons un système en boucle fermée défini par un opérateur de dimension infinie, et des opérateurs bornés d'observation et de contrôle. Supposons que ce système est exponentiellement stable. L'objet de cet exposé est de proposer un cadre et une loi d'échantillonnage de la boucle de commande de manière à ne mettre à jour le contrôle du système qu'à des instants pertinents pour conserver la propriété de stabilité exponentielle en boucle fermée. L'approche, basée sur une fonctionnelle de Lyapunov appropriée, permet de démontrer les résultats de bien posé, d'absence de point d'accumulation de la mise à jour et de stabilité attendus. Le cadre proposé englobe en particulier les équations de transport, des ondes, de KdV ou de Schrodinger.

## On Lyapunov functions and Input-to-State Stability of linear hyperbolic conservation laws and of Delay Difference Equations

*Delphine Bresch-Pietri*

This talk studies the Input-to-State stability of a linear Delay Difference Equation, subject to an additive external signal. We propose to use recent developments on necessary stability conditions for DDEs (Rocha Campos, Mondié and Di Loreto, IEEE TAC, 2018), to build a coercive Input-to-State Stability Lyapunov functional. This allows us to state a converse ISS Lyapunov theorem for this class of system. Then, exploiting the structural equivalence between DDEs and linear hyperbolic systems of conservation laws, we formulate a converse Lyapunov theorem for 1-D linear hyperbolic conservation laws. The corresponding Lyapunov function is explicit and does not require dissipative boundary conditions, contrary to the famous one proposed by Jean-Michel Coron and coworkers. This talk is based on a joint work with Jean Auriol (CNRS, L2S)

## Sur le temps de la contrôlabilité locale pour un systèmes de contrôle de KdV et pour un bac d'eau

*Jean-Michel Coron*

Nous nous intéressons à la contrôlabilité locale de l'équation non linéaire de Korteweg-de Vries (KdV) avec des conditions aux limites de Dirichlet en utilisant le contrôle au limite de Neumann à droite. Lionel Rosier a montré en 1997 que ce système KdV est contrôlable localement en petit temps pour toutes les longueurs non critiques et que l'espace non contrôlable du système linéarisé est de dimension finie lorsque la longueur est critique. Concernant les longueurs critiques, Emmanuelle Crépeau et moi avons montré en 2004 que le même résultat est valable lorsque l'espace incontrôlable du système linéarisé est de dimension 1, et plus tard Eduardo Cerpa en 2007, puis Eduardo Cerpa et Emmanuelle Crépeau en 2009 ont établi que la contrôlabilité locale est valable en temps grand pour toutes les autres longueurs critiques. Nous montrons que, pour une grande classe explicite de longueurs critiques, le système non linéaire de KdV n'est pas contrôlable localement en petit temps. Nous montrons également que pour un bac d'eau modélisé par les équations de Saint-Venant, il faut plus de temps pour la contrôlabilité entre états d'équilibre pour les systèmes non linéaires que pour le système linéarisé. Ces deux résultats sont des travaux en collaboration avec Armand Koenig et Hoai-Minh Nguyen.

## Reachable spaces for heat equations with lower order terms

*Sylvain Ervedoza*

The goal of this talk is to explain how perturbative arguments can be applied to derive a sharp description of the reachable space for heat equations having lower order terms. The main result I will present is the following one. Let us consider an abstract system  $y' = Ay + Bu$ , where  $A$  is an operator generating a  $C^0$  semigroup  $(\exp(tA))_{t \geq 0}$  on a Hilbert space  $X$ , and  $B$  is a control operator, for instance a linear operator from an Hilbert space  $U$  to  $X$ , and let us assume that this system is null-controllable in  $X$  in any positive time. Then, setting  $\mathcal{R}$  for the reachable set of the system (that is all the states that can be achieved by  $y$  solution of  $y' = Ay + Bu$ ,  $y(0) = 0$ ), the restriction of  $(\exp(tA))_{t \geq 0}$  to  $\mathcal{R}$  forms a  $C^0$  semigroup on  $\mathcal{R}$ . Accordingly, the system  $y' = Ay + Bu$  is exactly controllable on  $\mathcal{R}$ , and one can then perform classical perturbative arguments to handle lower order terms, as I will explain on a few examples. This talk is based on a joint work with Kévin Le Balc'h (INRIA Paris) and Marius Tucsnak (Bordeaux).

## Prescribed-time control for a class of semilinear hyperbolic PDE-ODE systems

*Nicolas Espitia*

In this talk, we will discuss recent results on prescribed-time control design for some classes of PDEs. To that end, we will first give an overview of non-asymptotic concepts such as finite, fixed, and prescribed time stabilization for both finite and infinite-dimensional systems (in particular 1D linear hyperbolic and parabolic PDEs). We will then present the control design (relying on prediction-based techniques) for a class of semilinear hyperbolic PDE which is bidirectionally connected with a nonlinear ODE. The approach uses a coordinate transformation to map the nonlinear system into a form suitable for analysis and control. As we will see, the transformation is based on predictions of system trajectories, which can be obtained by solving a general nonlinear Volterra integro-differential equation. The resulting prediction-based controller stabilizes the hyperbolic PDE-ODE system in a prescribed time.

## Control design for distributed parameter systems – the port Hamiltonian approach.

*Yann Le Gorrec*

This talk is concerned with the control of distributed parameter systems defined on a 1D spatial domain using the port Hamiltonian framework. We consider two different cases : when actuators and sensors are located within the spatial domain and when the actuator is situated at the boundary of the spatial domain, leading to a boundary control system (BCS). In the first case we show how dynamic extensions and structural invariants can be used to change the internal properties of the system when the system is fully actuated, and how it can be done in an approximate way when the system is actuated using piecewise continuous actuators stemming from the use of patches. Asymptotic stability is achieved using damping injection. In the boundary-controlled case we show how the closed loop energy function can be partially shaped, modifying the minimum and a part of the shape of this function and how damping injection can be used to guarantee asymptotic convergence. We end with some extensions of the proposed results to irreversible thermodynamic systems.

## Lie brackets : the good, the bad and the ugly

*Frédéric Marbach*

This talk will survey old and recent results on the local controllability of control-affine systems, focusing on results stated using Lie brackets of the vector fields defining the dynamics. In particular, we will try to understand what "good" and "bad" Lie brackets can mean in this context. Eventually, I will argue that some of them could also be classified as "ugly".

## Controllability of hyperbolic systems

*Hoai-Minh Nguyen*

In this talk, I will review results on the null-controllability, exact controllability of hyperbolic systems in one dimensional space for the optimal time. Both time-independent and time-dependent coefficients are discussed. Interesting and surprising phenomena occur. There is a finite gap for the optimal time in these cases even the time is involved only in the zero-order coefficients and even these coefficients are infinitely smooth. When the analyticity in time of the coefficients enters, the gap disappears. This is based on joint work with Jean-Michel Coron.

## **Stabilization of Nonlinear PDE by Means of Nonlinear Boundary Controls**

*Christophe Prieur*

In this presentation, the focus will be done on the design of boundary controls for distributed parameter systems, described by linear and nonlinear partial differential equations. Saturated controllers will be discussed in this talk as those modeling feedback laws in presence of amplitude constraints. We will review some techniques for the stability analysis and the derivations of design conditions for various PDEs as parabolic and hyperbolic ones. An application in nuclear fusion will conclude this lecture.

## **Stability analysis of sampled-data hyperbolic systems**

*Ying Tang*

In this talk, we focus on the stability analysis for a class of linear hyperbolic systems with distributed sampled-data control. A generic case with controllers discretized in both time and space has been addressed based on Lyapunov-Razumikhin techniques. Moreover, exponential stability for sampled-data (in time) hyperbolic systems has been established by a hybrid approach.

## **Frequency dependent Hautus conditions and perturbations for dispersive systems**

*Marius Tucsnak*

We show that, in many situations of interest, control/observation systems described by Schrodinger or Kirhhoff plate type equations satisfy a strong version of the classical Hautus condition, in which the constant yielding the exact observability time is replaced by a function of the frequency which becomes arbitrarily small when the frequency grows to infinity. We next use this to show that the exact observability property is preserved when the generator is perturbed by bounded (but not necessarily compact) operators, provided that they satisfy a standard unique continuation condition. Finally, we use the above results to derive local exact controllability for systems describing nonlinear plate vibrations.